Quantum Complexity and Entanglement

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Context: the q-machine

- A set of states $\{|\eta_k(L)\rangle\}$:
  $|\eta_j(L)\rangle = \sum_{w^L \in |\mathbf{A}|^L} \sum_{\sigma_k \in \mathbf{S}} \sqrt{Pr(w^L, \sigma_k|\sigma_j)}|w^L\rangle|\sigma_k\rangle$
- Q-machine's initial state:
  $\rho = \sum_i \pi_i |\eta_i(L)\rangle\langle\eta_i(L)|$

What can the Q give us?

- We've seen $C_q \leq C_\mu$
- But now we have the full machinery of a quantum system, what else can it give us?

Entanglement

- An exclusively quantum resource.
- It makes quantum information and quantum computation a lot more interesting.

Biased Coins Process

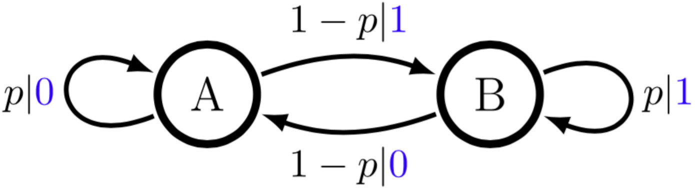
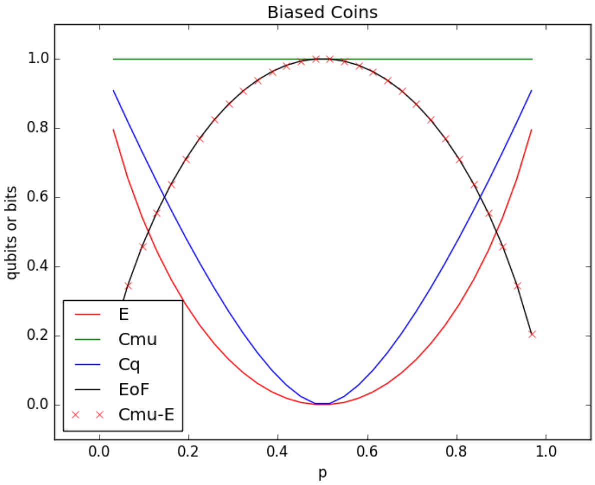


Figure: Biased Coins

Biased Coins Process



Measurements of Entanglement

- Can we actually measure this thing?
- How about for bipartite systems?
  - Pure states ✓
  - Mixed states ... ✓?

Entanglement of a pure state

- A quantum system composed of two parts labeled A and B
- The entanglement of a pure state Φ is:

$$E(\Phi) = S(Tr_A|\Phi\rangle\langle\Phi|) = S(Tr_B|\Phi\rangle\langle\Phi|)$$

- But what does it mean?

Bell States

$$|e_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|e_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|e_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|e_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Entanglement of Formation

- Take any of the Bell states (e.g. the singlet) as the standard state.
- Imagine you are given a large number m of this Bell state. By means of a (LOCC) protocol you can create n copies of state $|\Phi\rangle$.
- Entanglement of formation is the minimum ratio m/n in the limit of large n.
- Schematically:

$$nE(\Phi) \times Bell \rightarrow n \times |\Phi\rangle$$

EoF for mixed states

- A mixed state: $\rho = \sum_{j=1}^{N} p_j |\Phi_j\rangle\langle\Phi_j|$
- So we could say:

$$E(\rho) = \sum_j p_j E(\Phi_j)$$

...but not really

EoF for mixed states

The following mixed state:

$$\rho = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

Can be a mixture of:

$$|00\rangle$$

$$|11\rangle$$

or a mixture of:

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

EoF for mixed states

$$E(\rho) = inf \sum_j p_j E(\Phi_j)$$

For a pair of qubits:

$$E(\rho) = \epsilon(C(\rho))$$

Where C is the concurrence and:

$$\epsilon(C) = h(\frac{1+\sqrt{1-C^2}}{2})$$

$$h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$$

Concurrence

- The concurrence can be regarded as a measure of entanglement in its own right.
- For a pure state:

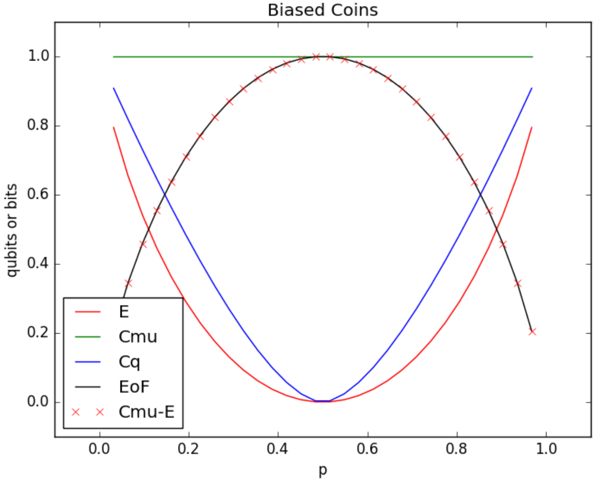
$$C(\Phi) = |\langle\Phi|\Psi\rangle|$$

With: $|\Psi\rangle = \sigma_y|\Phi^*\rangle$

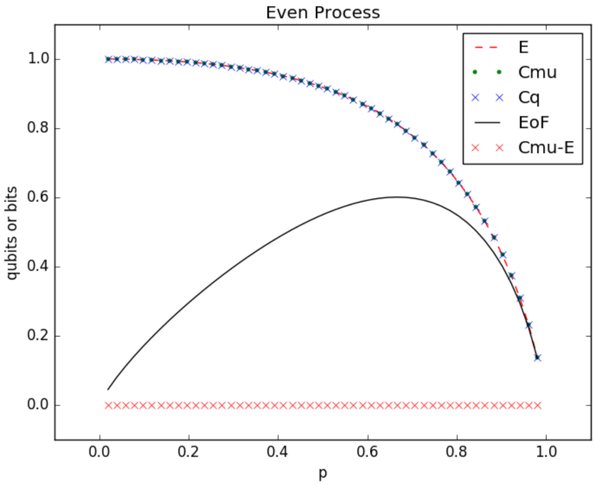
- For a mixed state

$$C(\rho) = inf \sum p_j C(\Phi_j)$$

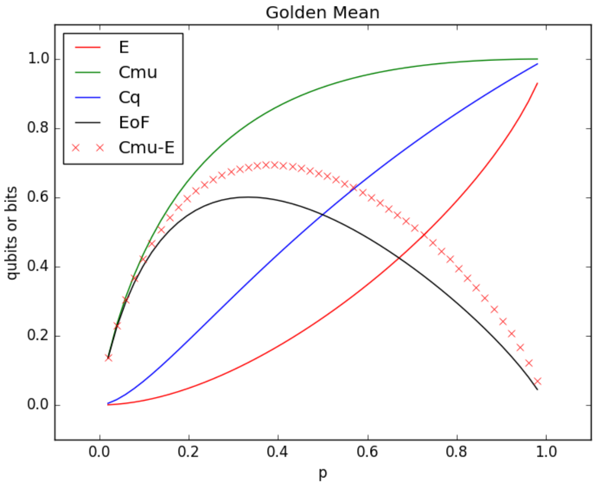
Biased Coins Process

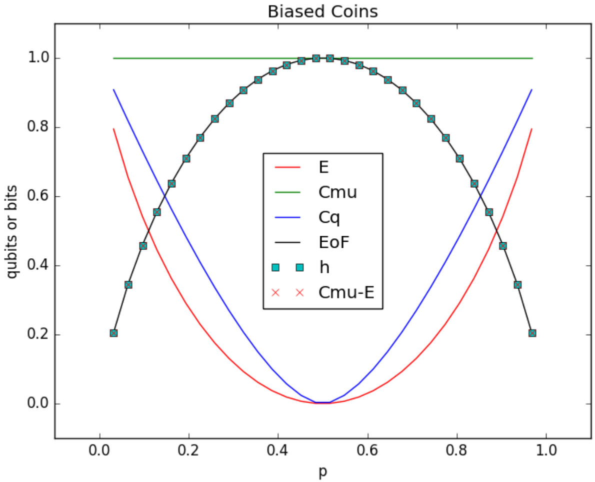


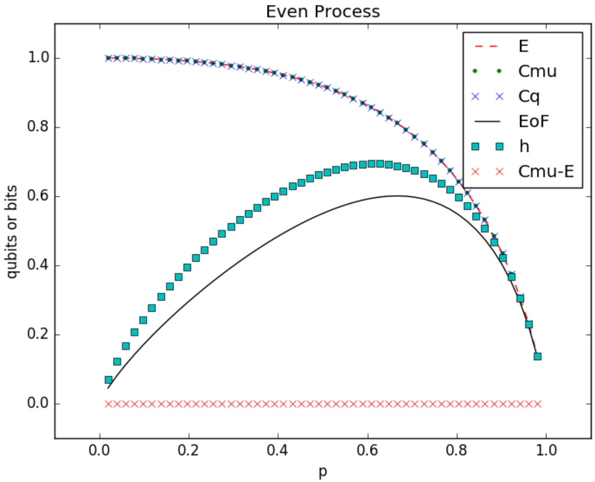
Even Process



Golden Mean Process




Biased Coins
qubits or bits
p
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
E
Cmu
Cq
EoF
h
Cmu-E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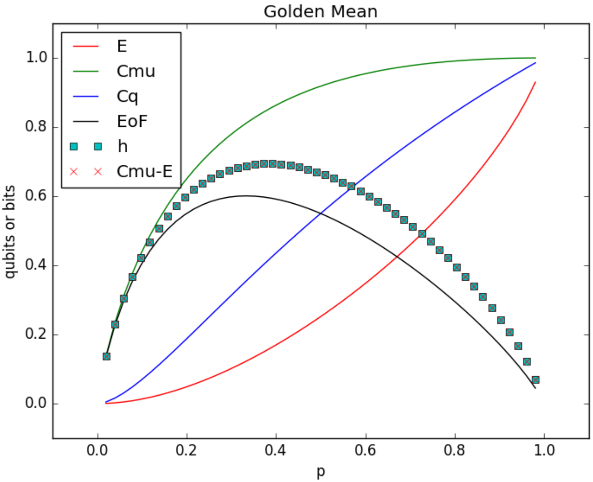

Golden Mean
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Some considerations about EoF

- EoF has several advantages:
  - It has a very sound interpretation
  - It reduces to the standard measure of entanglement for pure states
  - Formula for 2 qubits
- And some disadvantages:
  - Not trivial for other size systems
  - Ratio problem

Towards Interpretation

- As opposed to the general case of two qubits, our states have several constraints.
- Can we have maximally entangled states?
- One has to consider the fact that the two spaces look the same but are not the same.
- What does it tell us about the process? Can we do something with this?
- Other measurements?
- How can we handle larger Hilbert spaces?
- ???

References

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Thank You!